INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids 2008; **57**:1695–1708
Published online 5 December 2007 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.1672

One-dimensional simulation of supercritical flow at a confluence by means of a nonlinear junction model applied with the RKDG2 method

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SUMMARY

We investigate the one-dimensional computation of supercritical open-channel flows at a combining junction. In such situations, the network system is composed of channel segments arranged in a branching configuration, with individual channel segments connected at a junction. Therefore, two important issues have to be addressed: (a) the numerical solution in branches, and (b) the internal boundary conditions treatment at the junction. Going from the advantageous literature supports of RKDG methods to a particular investigation for a supercritical benchmark, the second-order Runge–Kutta discontinuous Galerkin (RKDG2) scheme is selected to compute the water flow in branches. For the internal boundary handling, we propose a new approach by incorporating the nonlinear model derived from the conservation of the momentum through the junction. The nonlinear junction model was evaluated against available experiments and then applied to compute the junction internal boundary treatment for steady and unsteady flow applications. Finally, a combining flow problem is defined and simulated by the proposed framework and results are illustrated for many choices of junction angles. Copyright © 2007 John Wiley & Sons, Ltd.

Received 19 July 2007; Revised 8 October 2007; Accepted 14 October 2007

KEY WORDS: confluence; internal boundary conditions; RKDG2; supercritical; steady and transient

1. INTRODUCTION

Routing steady and unsteady flows in open-channels has been an important topic for the computational fluid dynamics community due to the wide existence of hydraulic engineering applications. Extensive research has been performed in this area during the last two decades, and different

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numerical schemes have been developed in the context of finite difference (FD), finite volume (FV) and finite element (FE) methods [1–11], though some of them deal with open-channel networks [12–20]. In the simulation of flows *via* networks of channels/pipes, a key issue is how to deal with junctions because they behave as internal boundary conditions that are parts of the solutions. Hence, the situation is more complex due to the need to solve the equations at channel junctions. In an attempt to avoid solving complicated nonlinear algorithms resulting from a fully dynamic approach, most modellers rather assume stages' equality at the junction following the energy equation approximation of Akan and Yen [21]. However, this concept is not recommended for the supercritical flow case. This paper aims to present a one-dimensional numerical FE-type method to simulate supercritical flow through a rectangular single-junction network by using a nonlinear model, based on physical principles, to solve the equations at the junction.

Supercritical junction flows were firstly examined by Bowers [22] and Schnitter *et al.* [23], who provided a partial solution to the problem. Behlke and Pritchett [24] analyzed supercritical junction flow in rectangular and trapezoidal channels. Angle of confluence considered was $\theta = 15^\circ$ and 45°, and the inflow Froude number ranged between 2 and 7. Gildea and Wong [25] presented design information on the concrete-lined open-channels and show how model test results can modify and improve proposed designs. Greated [26] studied supercritical flow in rectangular 60° open-channel simple junctions. The author considered horizontal channels with supercritical flow at relatively high Froude number (6–11) in all branches; the agreement between prediction and observation (obtained for channels of branch widths $B = 12.7\,\text{cm}$) is fair. Wong and Robles [27] analyzed flow characteristics for the major junction by the momentum principle. Results were verified by experimental tests. The experimental results of many junctions substantiated those calculated theoretically. The authors also reported criteria to design a supercritical flow junction, with very little wave formation and turbulence, leading to good flow characteristics at the junction. Rice [28] reported qualitative observations of the flow at the junction for two supercritical flows at a 60° angle, noting that a jump formed in the main channel when the lateral flow exceeded a certain value. Rice [29] analyzed rectangular channel junctions with angles of $\theta = 30^\circ, 60^\circ$ and 90°. The researcher presented the results of preliminary physical model tests designed to provide insight into open-channel junction design with supercritical flows and identify the pertinent variables that affect the flow behavior in the junction area. Hager [30] presented a comprehensive theoretical and experimental study of the junction for two supercritical flows in channels of equal widths and suggested a criterion for the formation of the hydraulic jump at the junction, his experiment referred to angles of $\theta = 45^\circ$ and 22.5°. Christodoulou [31] analyzed theoretically the condition for an hydraulic jump formation at a supercritical flow confluence. The author presented a one-dimensional analysis for rectangular channels to approximate criteria for the channels' discharge ratio in terms of the junction geometry and approaching flow conditions. The effect of the main factors, notably the Froude numbers and the channel's widths ratio, is determined. Experiments were carried out for junctions at a 90° and a 17° angle of a subcritical tributary flow and to a weakly supercritical main flow (the Froude number range was between 1.5 and 2.0). Schwalt and Hager [32] conducted an experimental study for a supercritical flow in unobstructed simple junction geometry. The authors extended test results which yield the simplified pattern of standing waves and the flow separation in the junction for angles $\theta > 15^\circ$. Transitions to choking flows, where the supercritical flow across the junction breaks down, were also examined. These studies were frequently restricted to junctions' theory and experimental setups. Mostly, they were not associated with the one-dimensional open-channel networks' numerical simulations for the handling of internal boundary conditions.

DOI: 10.1002/fld

FD methods such as the Preissmann scheme [17, 18] have been widely used in simulating flows in pipes/channels networks. Recently, a traditional implicit FE method [20] has been applied to open-channel networks and showed nearly identical results to that from the implicit Preissmann scheme. On the other hand, simulations of steady and unsteady flows in pipes/channels networks have also been reported with implicit [15] total-variation-diminishing (TVD) schemes and the relaxation model [5] with the IMEX explicit–implicit [12] time integration. However, applying explicit methods to simulate flows through a system of compound channels could prove advantageous by using a minimum of computational cells that leads to a reasonable simulation time cost without generating diffusions in the numerical solutions. Therefore, supported by relevant references reported in literature [8, 11, 33], the FE-type RKDG2 scheme is chosen for the approximation of the numerical solutions in channels. Furthermore, a supercritical benchmark having an analytical solution is selected [34] for the purpose of comparing the effectiveness of the RKDG2 model with two TVD-FV methods that were implemented with the same properties as RKDG2.

Herein, the topic of interest is how to handle the combination of flows at the junction in the supercritical case. Like many investigators, we utilize a decomposing approach that is similar to that of Schaffranek *et al.* [17]. A simple channel network is considered as a large system consisting of three subsystems (Figure 4), i.e. branches. Also, the system can be considered to be a main channel having a tributary lateral inflow. As Figure 4 indicates, finding the solution at the junction points means to solve for six unknowns (three depths and three discharges). According to the characteristics theory [35–38], as the flow is supercritical, four equations are available propagating numerical boundary conditions [37] to the end points of the channels upstream of the junction. By assuming flow continuity through the confluence, a fifth equation is provided. For completeness, one still needs an additional equation, the sixth nonlinear equation consists of the momentum conservation model projected into the direction of the main flow. By solving this system of equations, junction solutions are obtained (and therefore the inflow boundary conditions of the downstream branch).

In this paper, we focus on the resolution of the junction problem for a simple confluence system. A thorough technique, for flow simulation through the junction system, is proposed based on two essential issues: (a) numerical solution of the shallow water equations in the branches by means of the RKDG2 scheme and (b) evaluation of the nonlinear junction model with respect to available experiments, and its application with the RKDG2 scheme as a combining model involved to perform the solving for the junction's variables. Finally, a hypothetical simple confluence system, involving steady and transient flows, is defined and simulated in order to illustrate the effectiveness of the proposed method. The contents of this paper are organized as follows: firstly, we discuss the analysis of the solution in channel branches. Secondly, we explain the boundary conditions' treatment. Finally, numerical results of a particularly steady and transient supercritical junction flow are illustrated.

2. ANALYSIS OF THE FLOW IN BRANCHES

2.1. Shallow water equations

The St. Venant equations [39] are widely used in the modelling of open-channel flows. Coupled with a reliable, accurate numerical solver, the equations provide a vital tool in the design of drainage and irrigation networks. In one space dimension, and for a wide prismatic channel with a

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rectangular cross-section, the continuity and momentum equations are expressed in the following conservative form:

$$U_t + f(U)_x = G(U) \tag{1}$$

where $U=[A;\, Q]^{\mathrm{T}}$ is the flow vector, $f(U)=[Q;\, Q^2/A+0.5\,gBh^2]^{\mathrm{T}}$ the flux vector, and $G(U)=[0;\, gA(S_0-S_{\mathrm{f}})]^{\mathrm{T}}$ the source terms vector. t represents time (s), x the longitudinal distance (m). $A=Bh$ is the wetted cross-sectional area (m^2), Q the flow discharge (m^3/s), g the acceleration due to gravity (m/s^2), B the channel bottom width (m) and h the water depth (m). $S_0=-\partial z/\partial x$ is the bed slope, where z designate the bottom elevation (m). $S_{\mathrm{f}}=Q^2n^2/(A^2R^{4/3})$ represents friction effects, where n denotes the Manning's roughness coefficient and $R=Bh/(B+2h)$ the hydraulic radius.

2.2. *RKDG2 scheme—an overview*

This subsection briefly describes the construction and implementation of the RKDG2 method for the one-dimensional shallow water equations. A channel segment is divided into N uniform cells $I_i=[x_{i-1/2}, x_{i+1/2}]$ where the points x_i are the centers of the cells, and $\Delta x = x_{i+1/2}-x_{i-1/2}$ the cell's size, assumed to be uniform. The proposed discretization is reported in detail in Reference [8]. We seek a local approximation (piecewise linear) U_h to U that belongs to the finite dimensional space $P^k(I_i)$ of polynomial in I_i of degree at most $k=1$ leading to second-order accuracy in space. Therefore, system (1) is multiplied by an arbitrary smooth function and integrated over I_i. Then, the flux term is integrated by part to obtain the weak formulation (see [8, 10, 40]). With the aim of decoupling the system, we adopt the Legendre polynomials as a local basis function over I_i. Therefore, the approximation of the solution $U_h(x,t)$ over each cell I_i can be expressed as

$$U_h(x,t)|_{I_i} = U_i^0(t) + 2U_i^1(t)(x-x_i)/\Delta x, \quad \forall x \in I_i \tag{2}$$

At each time step, we have to solve for $\{U^0(t), U^1(t)\}$ going from the projected initial condition,

$$\begin{aligned} U_i^0(0) &= U(x_i,0) = U_0(x_i) \\ U_i^1(0) &= \sqrt{3}/2[U_0(x_i+\Delta x\sqrt{3}/6) - U_0(x_i-\Delta x\sqrt{3}/6)] \end{aligned} \tag{3}$$

and, for the update of the degrees of freedom, one has to proceed as follows:

$$\mathrm{d}U_i^{0,1}/\mathrm{d}t = L_{0,1}(U^0, U^1) \tag{4}$$

L_0 and L_1 are the DG space operators having the following structure:

$$\begin{aligned} L_0(U^0,U^1) &= -1/\Delta x[\tilde{f}(\hat{U}^+_{i+1/2}, \hat{U}^-_{i+1/2}) - \tilde{f}(\hat{U}^+_{i-1/2}, \hat{U}^-_{i-1/2}) - \Delta x G(U_i^0)] \\ L_1(U^0,U^1) &= -3/\Delta x[\tilde{f}(\hat{U}^+_{i+1/2}, \hat{U}^-_{i+1/2}) + \tilde{f}(\hat{U}^+_{i-1/2}, \hat{U}^-_{i-1/2}) - f(U_i^0 - U_i^1/\sqrt{3}) \\ &\quad - f(U_i^0 + U_i^1/\sqrt{3}) - \Delta x\sqrt{3}/6(G(U_i^0+U_i^1/\sqrt{3}) - G(U_i^0 - U_i^1/\sqrt{3}))] \end{aligned} \tag{5}$$

Since U_h is discontinuous at the points $x_{i\pm 1/2}$, the ambiguity present in the terms involving the nonlinear fluxes must be replaced by numerical fluxes [41] that depend on the two different values of U_h at the interfaces. It is worth to point out that a slope limiting procedure must be performed

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to U_h before applying the numerical flux function $\tilde{f}(U_R, U_L)$. Therefore, a limiter function [40] is applied to maintain the non-oscillatory property of the RKDG2 method, namely,

$$U_h^{\lim}|_{I_i} = U_i^0 + 2(x - x_i)/\Delta x \min\mathrm{mod}(U_i^1, U_i^0 - U_{i-1}^0, U_{i+1}^0 - U_i^0) \tag{6}$$

where

$$\min\mathrm{mod}(a_1, a_2, a_3) = \begin{cases} s\min(|a_1|, |a_2|, |a_3|) & \text{if } s = \mathrm{sign}(a_1) = \mathrm{sign}(a_2) = \mathrm{sign}(a_3) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Following Kesserwani *et al.* [41], the approximate Riemann solver of Roe and Pike [42], whereby the computation of necessary items does not require the Roe average Jacobian matrix, is one of the suitable choices for the numerical flux function adapted with RKDG methods.

To render second-order accuracy in time, the second space order semi-discrete scheme (4) is discretized in time by a two-step nonlinearly stable RK time mechanism with a CFL number equal to 0.333 for stability requirements [40].

2.3. Numerical model selection

Up to now, a variety of numerical methods for solving open-channel flows have been proposed by many researchers. Some are based on flux splitting (upwind) techniques [1] and some others are variants the Lax–Wendroff nonlinear flux [7, 38]. A comparison of the performance of several TVD methods can be found in [6]. Nevertheless, many recent innovative approaches have appeared for solving the shallow water equations. Burguete and García-Navarro [2] pioneered a new approach for constructing high-resolution TVD schemes and considered application to shallow water flow. Delis and Katsaounis [5] applied the relaxation scheme for the shallow water equations. The authors presented a new approach for incorporating source terms in the relaxation model. Črnjarić-Žic *et al.* [3] extended FV weighted essentially non-oscillatory (WENO) schemes and central WENO schemes by particularly considering open-channel flow applications. Crossely and Wright [4] applied two local time stepping (LTS) strategies to simulate the one-dimensional unsteady water flows. Mohammadian *et al.* [9] carried forward an extension to the non-conservative method of characteristics (MOC). By using a proper interpolation function, the MOC is rendered conservative and can handle challenging tests for the one-dimensional open-channel flow (dam-break type, transcritical flows).

RKDG methods have become popular and are used in many fields of applications. However, when applied to simulate shallow water flows, the numerical model merely considers frictionless and/or horizontal channel flows [10, 11]. Recently, Kesserwani *et al.* [8] presented the simulation of discontinuous water flows by applying the RKDG2 scheme to the full conservative form of the St. Venant equations. The algorithm proved to be effective compared with a traditional TVD-FV model implemented with same properties as RKDG2, i.e. second-order accuracy, Roe Riemann solver and pointwise treatment of the source terms. Furthermore, the RKDG2 method led to positive results when coupled with the MOC to handle internal and external boundary conditions.

In this work, the RKDG2 scheme will be used for solving the St. Venant equations in branches. In fact, many existing references point out the advantages of this class of FE methods. Zhou *et al.* [33] presented a quantitative comparison of a third-order FV-WENO method and third-order DG scheme, all with a third-order RK time discretization. The researchers' results show that to achieve the same error magnitude, RKDG schemes usually require fewer spatial mesh points and less CPU-time cost than FV-WENO methods, although a CFL number of 0.6 was chosen for

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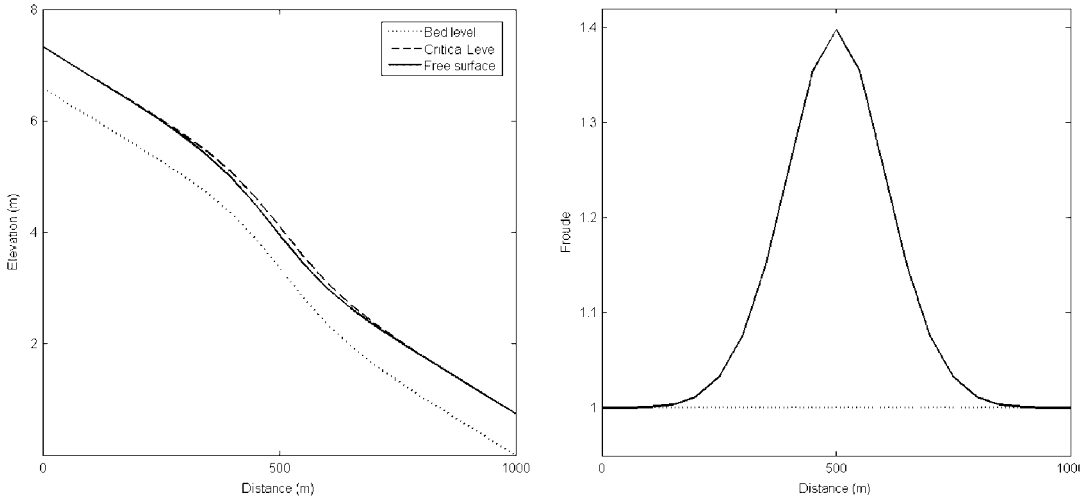

Figure 1. Free surface and Froude number profiles for the supercritical flow test.

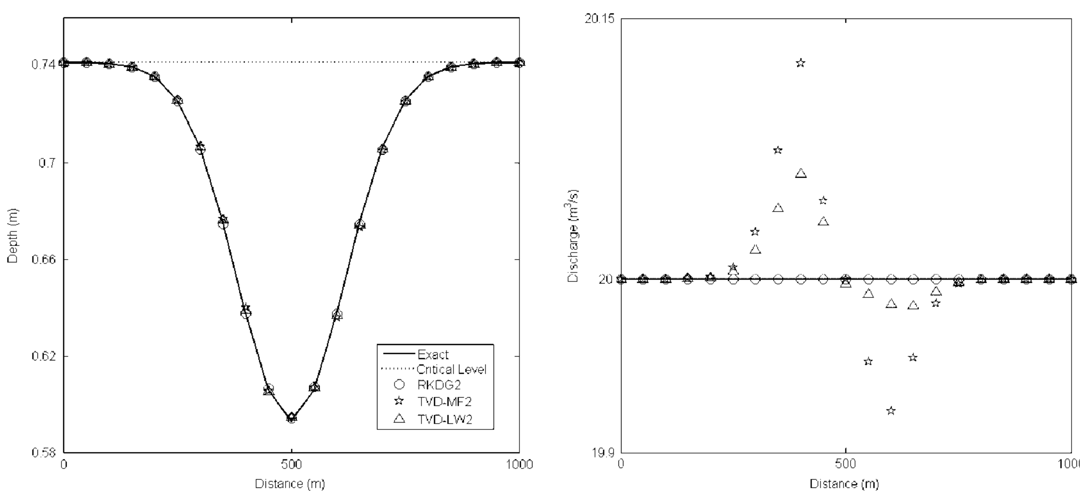

Figure 2. Comparison of the RKDG2 scheme with the results of two traditional TVD schemes implemented with the same properties as RKDG2.

WENO and a CFL number of 0.2 was chosen for RKDG. Xing and Shu [11] considered high-order WENO schemes and RKDG schemes. The authors claimed that the RKDG method is by far the simplest approach to obtain a well-balanced numerical scheme.

As this survey deals with supercritical flow, we consider a numerical investigation comparing the RKDG2 results with the performance of traditional second-order TVD-FV models. Two second-order Roe extension schemes are selected. One is based on the upwind modified approach flux of Harten [6] (MF2), and the other model (LW2) come from the adjustment of the Lax–Wendroff flux (weighted by a flux limiter function) to enforce the TVD property [38]. In this essay, the TVD-MF2 was computerized as presented in Delis and Skeels [6], whereas, the TVD-LW2 is implemented according to the formulation presented in Crossely and Wright [4]. However, two important differences have to be noted for the TVD-LW2 model: (a) the minmod slope limiter is used, and (b) the source terms vector is treated by a pointwise approximation. The comparison is performed on a steady supercritical benchmark, with analytical solution, chosen from MacDonald *et al.* [34]. The discharge was equal to $20\,\mathrm{m}^3/\mathrm{s}$; a spatially varied bottom slope is involved as well as friction effects. For more details, the flow patterns for this test are shown in Figure 1 while Figure 2 displays the numerical results achieved by the computation of the FE and the two FV solvers using only 21 cells. A reasonable approximation to the depth profiles was performed by all the schemes. However, analyzing the middle portion in the discharge plots, where the Froude ranged between 1.05 and 1.4, one can spot a poor preservation of the discharge achieved by the FV models. On the other hand, a perfect agreement with the analytical solution was observed for the RKDG2 scheme.

3. BOUNDARY CONDITIONS

3.1. External

The conventional MOC has been used for a long time in open-channels and pipe flows. It is based on non-conservative equations; hence, it cannot be used directly for solving discontinuous and transcritical flows [9]. On the other hand, this form provides insight into shallow water wave motion, which is not evident in the other forms. Thus, literature has widely dealt with the MOC as a starting point for boundary conditions. In the case of supercritical flow through a finite length channel branch, conditions at the boundaries are shown in Figure 3. As the flow is supercritical, the interval of dependence [35, 37, 38] of points on the upstream boundary in the $(x-t)$ plan is outside the length of the branch analyzed, which is entirely the opposite situation of the interval of dependence of points at the downstream boundary. Therefore, water depth and flow discharge must both be supplied at the inflow. This kind of boundary conditions can be called physical boundary conditions. At the downstream end, no boundary conditions are required because the interval of dependence falls within the length of the channel analyzed. Both characteristics should propagate

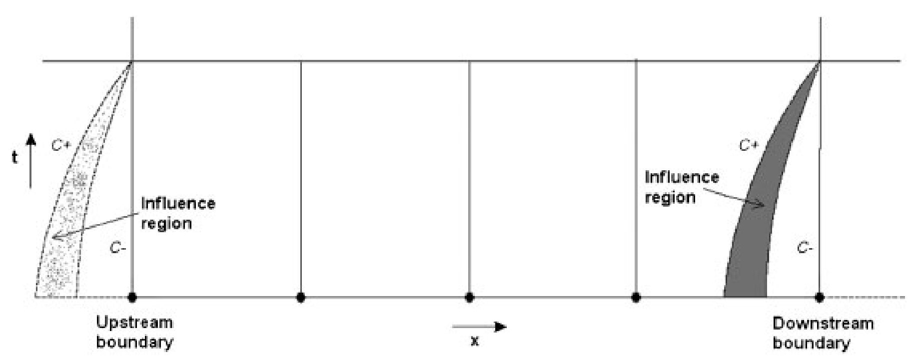

Figure 3. Characteristic curves in the $(x-t)$ plan for a supercritical flow regime.

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information from upstream to downstream. Such a kind of condition is referred to as numerical boundary conditions. Many interesting options for treating numerical boundary conditions in one-dimensional shallow water flow have been recently reported by Burguete *et al.* [37]. However, as the flow regime remains supercritical, the MOC is incorporated in the schemes to achieve numerical boundary conditions [36].

3.2. Internal

In this section, a new approach is presented for handling the combination of flows at the junction. Generally, to find the solution at the internal points enclosing the junction (Figure 4), one has to solve for six unknowns, three discharges Q_u, Q_L and Q_d and three water levels h_u, h_L and h_d. The subscripts 'u', 'L' and 'd' indicate the flow parameters at the junction relative, respectively, to the upstream, lateral and downstream branches. Therefore, one must have six concerned equations. As mentioned before, due to the supercritical nature of the flow, two equations for each of the upstream and lateral branches will be furnished, by using MOC, propagating the flow information to their downstream boundary points (in the form of linear equations, see [15, 36]). Assuming flow continuity through the confluence, a fifth equation is available ($Q_d = Q_u + Q_L$). Since our case is of a main flow influenced by a local lateral inflow, we provide a sixth equation derived from the momentum conservation through the junction projected into the direction of the main flow. Denoting by θ the junction angle and assuming a unity weight of water, hydrostatic pressure distribution and horizontal bed at the junction, the nonlinear conservation model can be expressed as:

$$\frac{Q_d^2}{gB_d h_d} + \frac{B_d h_d^2}{2} = \frac{Q_u^2}{gB_u h_u} + \frac{Q_L^2 \cos(\theta)}{gB_L h_L} + \frac{B_u h_u^2}{2} \tag{8}$$

The nonlinear conservation of momentum model has been reported by several publications. Rice [29] has used this equation for supercritical flow at a combining junction. Ramamurthy *et al.* [43]

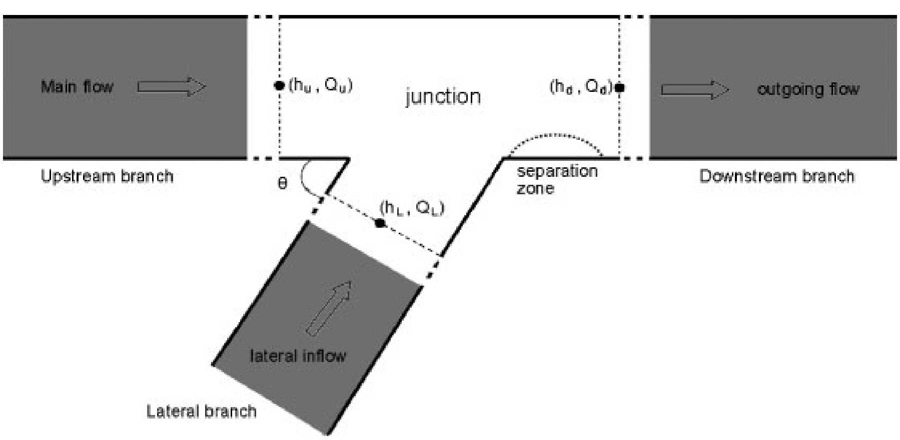

Figure 4. A schematic view of the one-dimensional junction problem, where the upstream and lateral flow contributions have to be propagated to the downstream channel by an internal boundary conditions treatment.

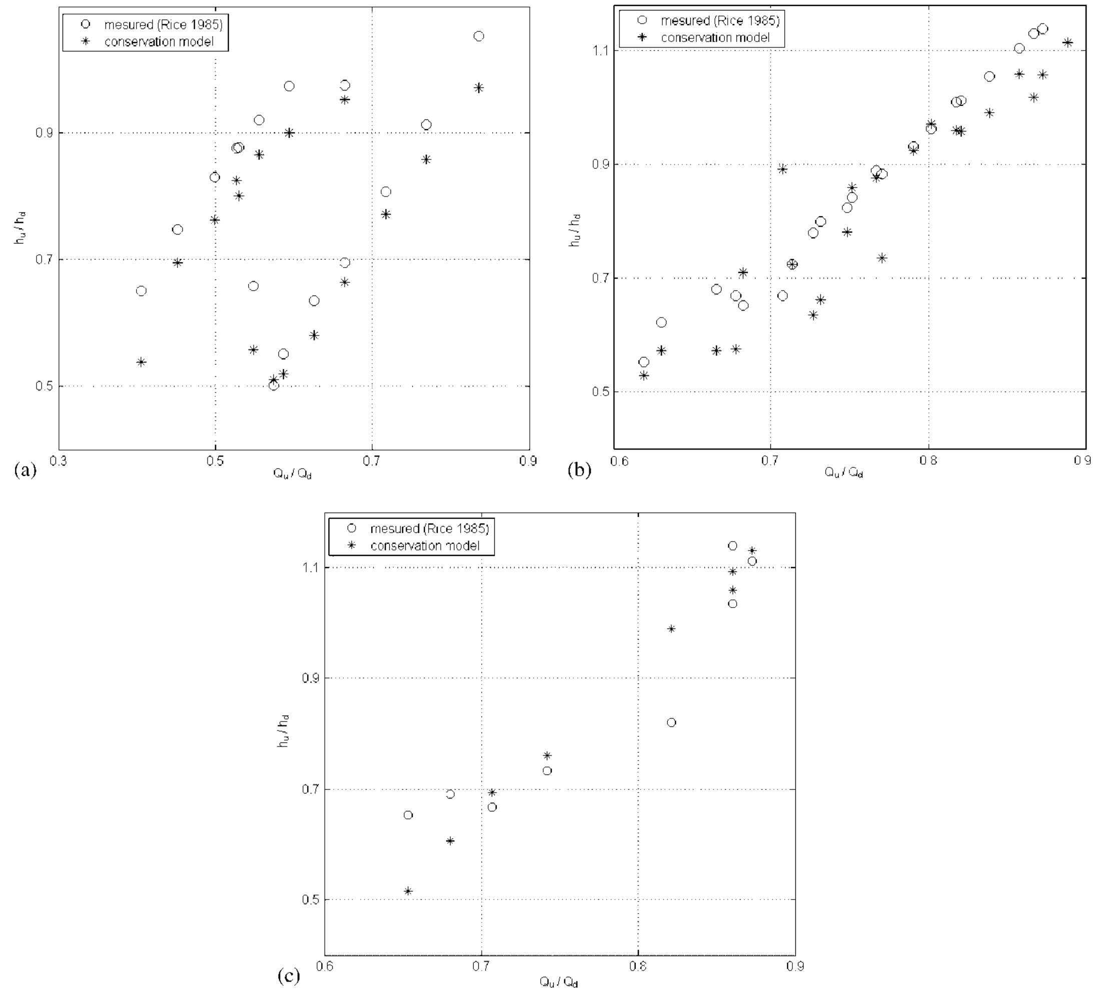

Figure 5. Evaluation of the nonlinear junction model with respect to experimental data: (a) $\theta = 30°$; (b) $\theta = 60°$; and (c) $\theta = 90°$.

proposed a model for combining open-channel flow at a right-angled junction; his model was based on the same principles as Equation (8). Christodoulou [31] considered this equation as a basis to his theoretical consideration. A very similar concern as Equation (8) was also provided by Hsu *et al.* [44] for the pioneering of a model to predict subcritical flow at a junction. As literature attests, this nonlinear model is not new. However, using it to contend the internal boundary condition treatment in the supercritical flow case is the main contribution of this paper.

At each time step, the outgoing depth h_d is calculated by applying Newton–Raphson iterations to solve the conservation model introduced with the numerical downstream boundary conditions (Q_u, h_u, Q_L, h_L) of the upstream and lateral branches, in addition to the conserved discharge Q_d. Therefore, the conservation equation has been validated according to available experimental

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data [29]. Giving inputs to the junction model, a selection of Rice's [29] flow data (the upstream boundaries at the junction and the discharge downstream of the junction) is used to compare the predicted outgoing depths and the experimental depths. Figure 5 contains the plots of the results of the experimental data and the predicted data in functions of the upstream-to-downstream discharge ratios and the upstream-to-downstream depth ratios. The performance of the conservation model was evaluated for three junction angles of $\theta = 30°, 60°$ and $90°$. Fair agreements between the predicted values and the experimental values are obtained.

4. NUMERICAL RESULTS FOR A CONFLUENCE SYSTEM

This section analyzes a fully supercritical flow computation throughout a simple confluence system. Supported by the previous discussions on the numerical methods (Section 2.3), and the internal boundary conditions (Section 3.2), the RKDG2 technique was applied to compute the free surface flow in branches in order to propagate, by the intermediary of the nonlinear junction model, the suitable flow data from the downstream edge of the main and lateral branches to the downstream waterway. Therefore, a steady case and a transitory case are defined and computed using 21 computational cells in each of the three branches. For both cases, junction angles of 30°, 45°, 60° and 90° were investigated on an hypothetical simple-network system composed of three rectangular channel branches. The main flow is affected by a local lateral inflow while remaining supercritical in the downstream channel. The main, lateral and downstream branches are, respectively, of 200 m length and 10 m width, 100 m length and 5 m width, and 600 m length and 10 m width. As discussed before, we have to specify flow inputs as physical upstream boundary conditions for each of the main and lateral branches. Thus, water depths of 0.3 and 0.2 m and discharges of 80 and 8 m^3/s were specified, respectively, at the upstream boundary points of the main and lateral branches. Friction terms and bed slopes have been taken into account in St. Venant's equations. The Manning's roughness coefficients are chosen equal to 0.02, 0.0125 and 0.0158 for the main, lateral and downstream branches, respectively. The bed slope of the main branch is taken equal to 2%, while a bed slope of 1% is set for the two others. The scheme has been left to convergence, and the steady-state profile of the water depth and the flow discharge for the three branches are illustrated in Figure 6. The upstream and lateral Froude numbers at the junction were, respectively, $F_u = 2.42$ and $F_L = 2.30$. Good mass conservation was transmitted, by the junction model, to the downstream branch. Furthermore, realistic junction angle effect was observed in the depths at the inflow of the downstream canal, while at the outflow we observe the physical convergence to the steady uniform depth.

Subsequently, by considering the steady-state profile as an initial condition, an unsteady case was defined by introducing continuous linear boundary condition at the inlet of the main and lateral canals. Owing to supercritical flow, both depth and discharge input hydrograph ($Q(t)$ and $h(t)$) are defined as physical boundary conditions for the main and lateral branches according to the following formulation:

$$U(t) = \begin{cases} U_{\min} + 2\dfrac{U_{\max} - U_{\min}}{t_2 - t_1}(t - t_1) & \text{if } t_1 \leqslant t \leqslant \dfrac{t_2 + t_1}{2} \\ U_{\min} - 2\dfrac{U_{\max} - U_{\min}}{t_2 - t_1}(t - t_2) & \text{if } \dfrac{t_2 + t_1}{2} \leqslant t \leqslant t_2 \\ U_{\min} & \text{otherwise} \end{cases} \tag{9}$$

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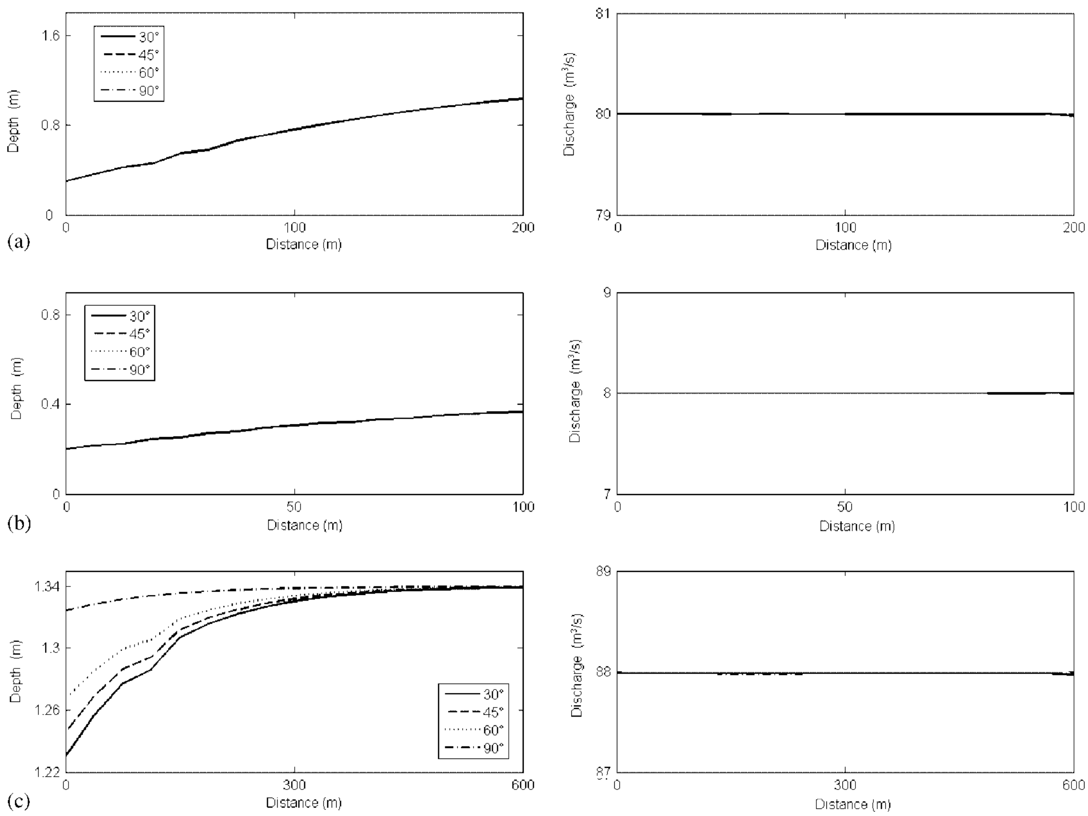

Figure 6. Steady-state (depths and discharges) profiles for the simulated confluence problem: (a) upstream branch; (b) lateral branch; and (c) downstream branch.

where $U(t)=[A(t);Q(t)]^{\mathrm{T}}$, $t_1=0, t_2=250$ and $0\leqslant t\leqslant 400$. $A(t)$ is the cross-sectional corresponding to the introduced depth limnigraph function $h(t)$. $U_{\min}$ and $U_{\max}$ are, respectively, the minimal and the maximal desired values at the inflow. At upstream and lateral inlets, going from the physical boundary conditions of the steady state as minimum values of the flow variables ($U_{\min,u}=[3;80]^{\mathrm{T}}$ and $U_{\min,L}=[1;8]^{\mathrm{T}}$), we linearly increase the flow variables to reach the maximum desired values ($U_{\max,u}=[4;100]^{\mathrm{T}}$ and $U_{\max,L}=[1.5;12]^{\mathrm{T}}$) and then linearly decrease them to bring back the previous minimum state. Following these conditions, the inflow Froude number at the upstream junction points varied between 2.36 and 2.60, while this number varied between 2.29 and 2.49 ($2.36\leqslant F_u\leqslant 2.60$ and $2.29\leqslant F_L\leqslant 2.49$) for the lateral inflow region at the junction. The outgoing flow remained supercritical, and the depth and discharge hydrograph simulated at the mid-point of the downstream channel are illustrated in Figure 7. All the investigated junction angles have close peaks. This is not surprising because, as noticed from Figure 6, at the mid-point of the downstream branch the flow will start to converge to the uniform state regardless of the variations of the junction angle. This small variation is also due to the relatively small contribution

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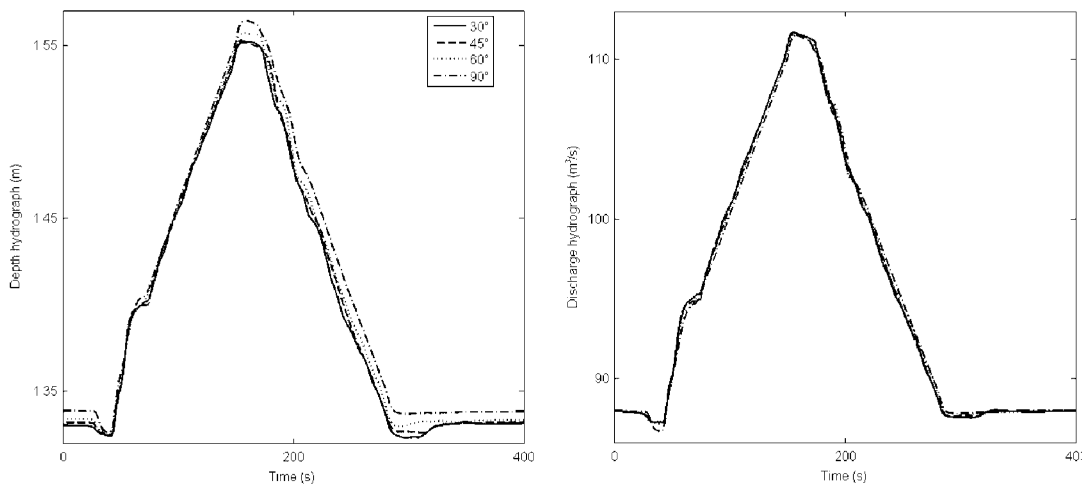

Figure 7. Hydrographs of the flow variables simulated at the mid-point of the outgoing branch.

of the lateral discharge. However, if we take a look at the free surface hydrographs, higher depth peaks were notable respecting the junction angle increase.

5. CONCLUSIONS

Over the past few years, the one-dimensional supercritical junction problem has received a little formal attention that mostly focuses on the junctions theory and experimental designs. Using the junction theoretical and experimental knowledge as means to handle internal boundary conditions, to propagate flow information from channel branches, is rarely examined. On the other hand, in simulating water flows through junctions, most modellers involve the concept of heads equality at the junction for completing the set of the boundary conditions treatment. However, this concept is not applicable in the supercritical case.

The RKDG method has seen a very rapid development due to its interest. The main emphasis of this method is that it can maintain the flow balance with no special treatment of the source terms and less computational cells (thus, less simulation time cost). Hence, dealing with this numerical scheme will ensure that the suitable information will be propagated and employed for solving the internal boundary conditions.

In this article, the interest was constrained to the numerical modelling of supercritical flow at a combining rectangular junction. The RKDG2 scheme was used for the computation of the water flow in channel branches. At the junction, the flow propagation was performed by involving a nonlinear model based on the momentum conservation. The model was evaluated against experiments and led to fair agreements. Subsequently, the model was applied to the simulation of steady and transient hydraulic junction problems. The defined problems involved a simple open-channels network with four cases of junction angles. The proposed framework has proven its utility for the considered applications: good mass conservation was observed in the steady discharge numerical

DOI: 10.1002/fld

model and a reliable outgoing water depth was established in all cases. However, the soundness of this approach for a more general flow modelling will be a future research project.

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DOI: 10.1002/fld